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# **Improving the Effectiveness of Weather-based Insurance: An Application of Copula Approach**

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IMPROVING THE EFFECTIVENESS OF WEATHER-BASED INSURANCE:  
AN APPLICATION OF COPULA APPROACH

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*Abstract*

The study develops the methodology for a copula-based weather index insurance rating. As the copula approach is better suited for modeling tail dependence than the standard linear correlation method, we suppose that copulas are more adequate for pricing a weather index insurance contract against extreme weather events. To capture the dependence structure in the left tail of the joint distribution of a weather variable and the farm yield, we employ the Gumbel survival copula. Our results indicate that, given the choice of an appropriate weather index to signal extreme drought occurrence, a copula-based weather insurance contract might provide higher risk reduction compared to a regression-based indemnification.

*Keywords: catastrophic insurance, weather index insurance, copula, insurance contract design*

*JEL classification: C18, Q14*

## INTRODUCTION

Weather index insurance has been considered as a valuable alternative for traditional crop insurance. The main advantage of the former is that it is better suited to combat asymmetric information problems, that is, adverse selection and moral hazard. An additional important advantage of weather-based insurance is that it considerably reduces transaction costs and thus allows a faster settlement of claims. The latter characteristic of weather index insurance makes it particularly relevant in the context of extreme weather event management, when the relief must be provided within a short period of time to a large number of affected farms.

Yet, as recent empirical evidence shows, despite substantial efforts of developing agencies such as the World Bank and governmental support to promote the market of weather index insurance in a number of developing countries, the demand for this instrument of risk reduction remains rather low (Cole et al. 2013; Mobarak and Rosenzweig 2012; Norton et al. 2011). Recent empirical studies examine and discuss different factors affecting farmers' demand for weather index insurance. In addition to factors evaluated in the context of traditional agricultural insurance such farm's socio-economic characteristics as risk aversion, level of production diversification etc., the literature discusses the effect of informal insurance (Mobarak and Rosenzweig 2012; Akter and Fatema 2011), basis risk (Barnett and Mahul 2010), and model prediction uncertainties (Bokusheva and Breustedt 2012).

Given this recent evidence, we analyze an option which would address the above mentioned issues and potentially increase risk reducing effectiveness of weather index insurance. We propose to design weather index insurance as an insurance against extreme events rather than an instrument to cope with moderate weather risks, as has been done in previous studies (Wang et al. 2013; Breustedt et al. 2008; Vedenov and Barnett 2004; Skees et al. 1997). In our view, this approach should positively influence the demand for weather index insurance as it could: (i) improve affordability of weather index insurance; (ii) reduce scope of basis risk; and (iii) improve predictive power of yield-weather models used for insurance contract rating.

(i) The presence of informal insurance might reduce demand for formal insurance against moderate risks due to a certain capacity of rural communities to share risk among their members (Mobarak and Rosenzweig 2012). However, financial reserves

within a rural community might be insufficient to cope with catastrophic risks, which usually have a systemic character and thus affect a large number of producers simultaneously. Accordingly, wealthier households of a community themselves might experience substantial yield losses in the case of a catastrophic event, which would affect their capacity and willingness to provide informal insurance to other members of the community. In this context, weather index insurance designed to cope with catastrophic risks might be better targeted to the needs of rural households, compared to insurance products insuring against moderate yield losses: moderate losses would be covered by using informal insurance, while extreme yield losses would be indemnified by a catastrophic weather index insurance.

(ii) The main reasons for basis risk are: (a) a low correlation of farm yields with a weather index due to the presence of other important risks, namely those beyond the hazard, to be insured by a particular weather-based insurance product (i.e., so-called loss-specific basis risk); (b) a low sensitivity of the farm yield to weather data of meteorological stations situated at a considerable distance to the farm (i.e., spatial basis risk); and (c) a low correlation between a weather index and a crop yield due to the timing of the occurrence of the insured event (i.e., temporal basis risk) (Skees et al. 2007). We presume that catastrophic weather index insurance against catastrophic events might be less affected by spatial basis risk as extreme events have a higher extent of spatial correlation. Compared to common weather index insurance, it would indemnify insured farms less frequently, but would potentially allow a more adequate coverage for yield losses in the case of extreme events.

(iii) Finally, we suppose that the application of methods appropriate for modeling extreme dependence, such as copulas could provide more robust estimates of yield dependence on weather and, thus, substantially limit the scope for model prediction uncertainties.

In our study, we evaluate the effectiveness of a catastrophic weather index insurance against drought by applying two alternative methods—the standard regression analysis and the copula approach. Most empirical analyses obtain estimates of the dependence of crop yields on a weather index by assuming a linear correlation structure in yield-weather dependence; i.e. they regard yield-weather distribution as a Gaussian multivariate distribution. This procedure has an important implication: the effect of the weather index on the yield conditional mean is assumed to be constant

over the whole distribution of the conditioning variable—the weather index. We argue that, when insuring against catastrophic events, the prediction of farm extreme yield losses can be done more accurately by employing the concept of tail dependence. In this study we develop and evaluate a copula-based approach for rating catastrophic weather index insurance.

In the empirical part of the analysis, we use the time series of wheat yield and weather variables for 47 large farms from two major grain-producing regions in Kazakhstan for the period from 1971 to 2010. We distinguish between three types of contracts. Weather index insurance is designed to pay indemnity whenever the weather index, chosen to indicate the extreme drought occurrence, falls below the first, second, and third deciles of its probability distribution. The risk-reducing effectiveness of insurance contracts is evaluated by employing three criteria: certainty equivalent, expected shortfall, and a modification of lower tail partial moment.

The remainder of the paper is structured as follows: Section 2 provides an overview of the methodology. Section 3 describes our data and empirical procedure. The study's preliminary results for a selected copula model and weather index are presented in Section 4. Conclusions are drawn in the final section.

## **METHODOLOGY**

Regression analysis is a standard tool used in empirical studies to estimate sensitivity of crop yields to a weather index when rating a weather index insurance contract. The use of regression analysis, however, introduces some restrictive assumptions on the joint distribution of variables employed in the model. In particular, the use of regression models limits the scope of the analysis to the Gaussian multivariate distributions and thus to the linear correlation dependence structure. The main implications of this procedure is that researchers implicitly assume that the sensitivity of crop yields to weather remains constant over the whole distribution of the yield variable as it is captured by the effect of weather on the yield conditional mean. Moreover, linear correlation is not adequate for representing dependency in the tails of multivariate distributions (McNeil et al. 2005). This quality of linear correlation questions its relevance for the assessment of extreme losses.

In our study, we design weather index insurance contracts by employing estimates of the linear regression, and compare them with estimates obtained from a copula-based approach, which we present below.

### *Copulas*

A copula allows marginal distributions to be linked together to form the joint distribution. A  $d$ -dimensional copula  $C(\mathbf{u}) = C(u_1, \dots, u_d)$  is a multivariate distribution function on  $[0, 1]^d$  with standard uniform marginal distributions (McNeil et al. 2005).

Sklar's theorem (1959) states that, if  $F$  is a joint distribution function with marginal distributions  $F_1, \dots, F_d$ , then there exists a copula  $C: [0, 1]^d \rightarrow [0, 1]$  such that for all  $x_1, \dots, x_d$  in  $\bar{R} = [-\infty, \infty]$ ,

$$F(x_1, \dots, x_d) = C(F_1(x_1), \dots, F_d(x_d)). \quad (1)$$

Therefore, according to Sklar's theorem, any continuous multivariate distribution can be uniquely described by two parts: the marginal distributions  $F_i$  and the multivariate dependence structure captured by the copula  $C$ . This definition explains the usefulness of copulas for modeling multivariate dependence.

In general, there are many families of copulas. Most important distinction is done between parametric and nonparametric (e.g., kernel) copulas. Empirical investigations primarily employ parametric copulas, which are better suited for simulation purposes. Parametric copulas consist of implicit and explicit families of copulas. Implicit copulas are defined by well-known multivariate distribution functions, e.g., the Gaussian copula and Student's  $t$  copula (McNeil et al. 2005). An important property of the explicit copulas is that they possess a simple closed form.

The best-known class of explicit copulas is Archimedean copulas, which comprises such copulas as Clayton, Gumbel, Frank, and Joe copulas. While the Gaussian, Student's  $t$ , and Frank copulas are elliptical copulas and assume radial symmetry, Clayton, Gumbel, and Joe copulas allow the modeling of joint distributions with asymmetric dependence structures. In particular, the Clayton copula exhibits strong left-tail dependence and weak right-tail dependence, whereas the Gumbel and Joe copulas demonstrate strong right-tail dependence and relatively weak left-tail

dependence. The definition of different families of copulas can be found in Nelsen (1999).

### *Copula model estimation*

The seasonality of agricultural production determines the length of time series available for empirical analyses. Regularly only one or two observations can be recorded within a calendar year. This fact reduces the scope for applying some methods, which require sufficiently long time series, to times series from agriculture. This issue concerns the application of the copula approach too. As the estimation of copula models necessitates a relatively large number of observations in the tails of a joint distribution, the estimation of the copula model for each single unit of investigation is rather problematic and might affect the model validity. To cope with this issue in our study, similar to Bokusheva (2012), we adopt the hierarchical Bayesian modeling framework, which allows model parameter estimation for each single study unit in a sample by employing the data of the whole sample population and considering its hierarchical structure.

To specify the Bayesian model for the copula estimation, we consider a bivariate vector  $(Y, W)$ , representing a crop yield and a weather variable, respectively. Then, the joint probability density function  $f(Y, W | \theta)$  for these two variables can be defined as:

$$f(Y, W | \theta) = c(F_Y(Y | \theta), F_W(W | \theta) | \theta) f_Y(Y | \theta) f_W(W | \theta), \quad (2)$$

where  $\theta$  is the vector of copula and marginal distributions' parameters,  $f$  and  $F$  denote a particular probability density and cumulative marginal distribution function, respectively, and  $c$  is a copula density. Then, regarding a sample of size  $N$  and length  $T$ , the respective likelihood function is given by:

$$L(Y, W | \theta) = \prod_1^{N \times T} c(F_Y(Y | \theta), F_W(W | \theta) | \theta) f_Y(Y | \theta) f_W(W | \theta) \quad (3)$$

Successively, the likelihood function is used to obtain the posterior distribution, defined as:

$$g(\theta | Y, W) = L(Y, W | \theta) g(\theta), \quad (4)$$

where  $g(\theta)$  is the prior distribution.

The copulas are usually estimated by a two-step procedure: In the first step, the parameters of marginal distributions are obtained by fitting a parametric distribution to the empirical data; In the second step, the parameters of the copula function are estimated by means of the Maximum Likelihood (ML) method. In this study, we also apply the two-step procedure. However, instead of the ML method, Markov chain Monte Carlo (MCMC) algorithms for Bayesian computation (Gamerman and Lopez 2006) were employed to obtain the joint posterior distributions of copula parameters.

#### *Insurance contract parameters*

To design and rate a weather index insurance contract, we employ the concept of marginal expected shortfall (MES), introduced in Acharya et al. (2012) and further elaborated on in Mainik and Schaanning (2012). MES is defined as the conditional expected shortfall of the target random variable ( $Y$ ) given that the conditioning variable ( $X$ ) exceeds its value at risk (VaR) at the  $\alpha$  confidence level, i.e.:

$$MES_{\alpha} Y|X = E\left(Y|X \leq VaR_{1-\alpha}(X)\right), \quad (4)$$

where  $E$  is the expectation operator.

We assume that the dependence structure between crop yields and a vegetation index might be stronger in the left tail than in the center or the right tail of the joint distribution. Thus, in our analysis we suggest focusing on the MES of the yield  $\tilde{\mu}^*$  conditioned on the realization of the weather index  $W$  below some specified quantiles. Then, given that  $W$  falls below a predetermined critical level, e.g., its  $\alpha$  quantile  $q_{\alpha}(W)$ , it is measured as:

$$\tilde{\mu}^* = MES|q_{\alpha}(W) = E\left(Y|W \leq q_{\alpha}(W)\right), \quad (5)$$

To determine  $\tilde{\mu}^*$  we have to define the conditional distribution of the yield variable, which is:

$$H_{Y|W=w}(y) = c_{G(Y)F(W=w)}(v) \Big|_{\substack{G(Y)=v, \\ F(w)=u}}, \quad (6)$$

where  $F(W)$  and  $G(Y)$  are the marginal distributions of the index and crop yield, respectively.



$H_{Y|W=w}(y)$  can be derived by taking the first derivative of the copula, which describes the joint distribution of the weather and yield variables, with respect to  $u$  corresponding with the weather variable marginal distribution as defined in (6), i.e.:

$$c_{G(Y)|F(W)=w}(v) = \frac{\partial}{\partial u} C(u, v) \Big|_{F(w)=u}, \quad (7)$$

Employing the definition in (7) and adjusting (6) to consider all realizations of the weather index below its  $VaR_\alpha$ , we rewrite the conditional distribution of the yield variable in (6) as follows:

$$c_{G(Y)|F(W) \leq 1-\alpha}(v) = \frac{1}{P(F(W) \leq 1-\alpha)} \int_{l=0}^{1-\alpha} \frac{\partial}{\partial u} C(u, v) \Big|_{\substack{F(w)=l \\ G(y)=v}} \partial l \quad (8)$$

The expression in (8) determines the conditional distribution of the yield variable in terms of a copula and the marginal distribution of the weather index. The integration of the expression in (7) over the whole yield marginal distribution  $[0,1]$ , and taking the inverse of the resulted expression, allows to determine the yield distribution quantile corresponding with  $\tilde{\mu}^*$ , i.e.:

$$\begin{aligned} \tilde{\mu}^* &= G^{\leftarrow} \left( \int_{k=0}^1 c_{G(Y)|F(W) \leq 1-\alpha}(s) dk \right) = \\ &G^{\leftarrow} \left( \frac{1}{P(F(W) \leq 1-\alpha)} \int_{k=0}^1 \int_{l=0}^{1-\alpha} \frac{\partial}{\partial u} C(u, v) \Big|_{\substack{F(w)=l \\ G(y)=k}} \partial l \partial k \right) \end{aligned} \quad (9)$$

where  $G^{\leftarrow}$  denotes the generalized inverse with respect to the yield marginal distribution function.

Once computed,  $\tilde{\mu}^*$  can be used to determine the indemnity value for each single realization of the weather index as the difference between the strike value of yield and corresponding value of the conditional expected yield, i.e.:

$$indemnity_t = [yield^{strike} - \mu_t^* | W_t \leq q_\alpha(W), 0 | W_t > q_\alpha(W)], \quad (10)$$

where  $t$  is the year index.

Successively, fair insurance premium can be calculated as expected indemnity value. Consequently, the insured yield was defined as:

$$y_t^{\text{insured}} = y_t + \text{indemnity}_t - \text{fair premium} . \quad (11)$$

The design of the insurance contract, based on the regression analysis, was done in accordance with the methodology presented by Skees et al. (1997). The threshold value of the weather index was set to the selected quantile values of the weather variable distribution. The insurance payout was conditioned on the same threshold values of the weather index in both approaches.

#### *Risk reduction evaluation*

The comparison of the effectiveness of weather index insurance contracts derived by the copula and regression approaches were done by employing three criteria: certainty equivalent, expected shortfall (also called conditional VaR), and lower partial moment of distribution.

The certainty equivalent (CE) was computed for insured and uninsured farm yields assuming negative exponential utility function, i.e.:

$$U(x) = 1 - \exp(-r_a x) , \quad (12)$$

where  $r_a$  is the Arrow-Pratt absolute risk aversion coefficient, the value of which was approximated by assuming a rather risk-averse decision-maker, as captured by the relative risk aversion coefficient ( $r_r$ ) equal to 2.0 (Hardaker et al. 2007) and setting the farmer's temporal wealth to the farm expected yield value.

The expected shortfall was calculated for three specified probability levels  $\alpha$  as:

$$ES = \frac{1}{1-\alpha} \int_{p=0}^{1-\alpha} q_p dp , \quad (13)$$

where  $q_\alpha$  is the yield distribution  $\alpha$ -quantile.

A modification of the lower partial moment (LPM) was computed as the expected value of squared negative deviations from the expected uninsured yield value for yield realizations below the third decile of the yield distribution:

$$LPM = \frac{1}{N} \sum_{t=1}^N [\min(y_t - \bar{y} | y_t < q_{0.3}, 0)]^2 \quad (14)$$

The evaluation of the risk-reducing effectiveness of insurance contracts was based on the relative risk reduction obtained by each type of insurance contract.

Based on the certainty equivalent criterion, the relative risk reduction of insured yields was calculated as:

$$RR^{CE} = \frac{CE^{insured} - CE^{uninsured}}{CE^{uninsured}}. \quad (15)$$

Analogously, the relative risk reductions based on two other criteria were computed as:

$$RR^{ES} = \frac{ES^{insured} - ES^{uninsured}}{CE^{uninsured}}, \text{ and} \quad (16)$$

$$RR^{LPM} = \frac{LPM^{uninsured} - LPM^{insured}}{LPM^{uninsured}}. \quad (17)$$

## DATA AND EMPIRICAL PROCEDURE

### *Data*

The study employs spring wheat yield data for 47 large farms from five counties located in two major grain-producing regions in Northern Kazakhstan. The farm yield time series were provided by the county statistical offices. The weather data was acquired from the National Hydro-Meteorological Agency of the Republic of Kazakhstan–Kazgydromet. The weather data originates from five weather stations situated in single study counties (one in each county) and comprises monthly records of cumulative precipitation and average daily temperature. Both yield and weather data cover the period from 1971 to 2010.

The yield time series were tested for the presence of structural breaks and were detrended by employing linear, and second and third degree polynomial time trend models. To enable consistent trend parameter estimates in the context of highly variable yield time series, we augmented trend models by adding a weather variable. Several weather indices, such as cumulative rainfall in different periods of the spring wheat vegetation, and modifications of the Selyaninov and Ped drought indices (Breustedt et al. 2007), were regarded as potential candidates for detecting extreme droughts. As the Ped drought index computed for two summer months (June and July) provided the highest levels of Pearson's and Kendall's rank correlations with undetrended farm yields on average for single counties, this index was selected to be

employed for the trend model estimation and to indicate the drought occurrence in the weather index insurance design. We specify the Ped drought index as:

$$Ped_t = \frac{R_t^{June-July} - \bar{R}}{\sigma_R} + \frac{P_t^{August-May} - \bar{P}}{\sigma_P} - \frac{T_t^{June-July} - \bar{T}}{\sigma_T}, \quad (18)$$

where  $t$  is the year index. The variables  $R$ ,  $P$ , and  $T$  are the cumulative rainfall, precipitation in millimeter, and average daily temperature in degree Celsius in an indicated sub-period, respectively;  $\bar{R}$ ,  $\bar{P}$ , and  $\bar{T}$  are their respective averages, and  $\sigma_R$ ,  $\sigma_P$ , and  $\sigma_T$  are the respective long-term standard deviations.

We distinguish between three potential threshold levels of the weather index to signal occurrence of an extreme drought; we specify them to correspond with the Ped drought index (PDI) realizations below the first, second, and third quantiles of its distribution.

Several distribution families were employed to fit yield and weather marginal distributions. The selection of the distribution was done on the basis of the Kolmogorov-Smirnov test. Accordingly, the Weibull distribution was chosen to model yields in the case of 15 farms; the gamma distribution provided the best fit for 14 farms; while logistic, normal, and log-normal distributions were selected for eight, eight, and two study farms, respectively.

### *Empirical Procedure*

As the estimation of copula models by means of hierarchical Bayesian models is a rather elaborate and time-consuming procedure, we reduce this part of the analysis by considering only one copula model—namely, the Gumbel survival copula. The choice of the copula model was done by estimating different copula models for single sample farms by employing the ML method<sup>1</sup> and evaluating their goodness of fit on the basis of the Cramer-von Mises statistics (Genest et al. 2009).

The survival Gumbel copula, defined as the Gumbel copula, applied to survival functions of marginal distributions<sup>2</sup> allows for measuring dependence in the left tail

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<sup>1</sup> To select an appropriate copula to be estimated by means of the hierarchical Bayesian model, we tested totally 6 copula models: the Gaussian copula, t-copula, Frank copula, Clayton copula, and Gumbel and Joe survival copulas. The Gumbel and Joe survival copulas provided best fits for a majority of the sample farms. The Gaussian copula corresponding with the linear correlation model showed best fit only for two sample farms.

<sup>2</sup> A survival function is defined as  $\bar{F}(x) = P(X \leq x) = 1 - F(x)$  (e.g. Nelsen 1999, p.32)

of the joint distribution. According to Nelsen (2005, p. 32), the survival copula is defined in terms of the corresponding copula model as:

$$\hat{C}(u, v) = u + v - 1 + C(1 - u, 1 - v) \quad (19)$$

Given the bivariate Gumbel copula

$$C_{\theta}^{Gu}(u, v, \theta) = \exp \left\{ - \left[ (-\ln u)^{\theta} + (-\ln v)^{\theta} \right]^{1/\theta} \right\} \text{ with } 1 \leq \theta < \infty, \quad (20)$$

where  $\theta$  is the copula dependence parameter, and the expression in (16), the conditional distribution of the weather variable in (6) can be defined in terms of the Gumbel survival copula as:

$$\begin{aligned} c_{v|u=u}(v) \Big|_u^v &= \frac{\partial \hat{C}^{Gu}(u, v)}{\partial u} = 1 + \frac{\partial C^{Gu}(1 - u, 1 - v)}{\partial u} \\ &= 1 - \frac{1}{1 - u} \exp \left\{ - \left[ (-\ln(1 - u))^{\theta} + (-\ln(1 - v))^{\theta} \right]^{\frac{1}{\theta}} \right\} \left[ (-\ln(1 - u))^{\theta} + (-\ln(1 - v))^{\theta} \right]^{\frac{1}{\theta} - 1} (-\ln(1 - u))^{\theta - 1} \end{aligned} \quad (21)$$

where  $(1 - u)$  and  $(1 - v)$  are the survival functions of the weather index and yield marginal distributions, respectively.

The estimation of the survival Gumbel copula in the framework of the Bayesian hierarchical modeling was done by deriving its density as:

$$\begin{aligned} \hat{c}^{Gu}(u, v) &= \frac{\partial^2 \hat{C}^{Gu}(u, v)}{\partial u \partial v} = \frac{\left[ (-\ln \bar{u})^{\theta} + (-\ln \bar{v})^{\theta} \right]^{\frac{1}{\theta}} + \theta - 1}{\bar{u} \bar{v}} \\ &\exp \left\{ - \left[ (-\ln \bar{u})^{\theta} + (-\ln \bar{v})^{\theta} \right]^{\frac{1}{\theta}} \right\} \left[ (-\ln \bar{u})^{\theta} + (-\ln \bar{v})^{\theta} \right]^{-2 + \frac{1}{\theta}} \left( \frac{1}{\bar{u}} \ln \bar{u} \right)^{-1 + \theta} (-\ln \bar{v})^{-1 + \theta} \end{aligned} \quad (22)$$

where  $\bar{u} = (1 - u)$  and  $\bar{v} = (1 - v)$ .

We employed the uniform distribution as the prior distribution of the copula dependence parameter to be estimated, i.e.,  $\theta \sim U(a, b)$ . This allowed us to easily account for the left-hand censoring of the dependence parameter of the Gumbel copula. We used non-informative prior distributions by defining  $a \sim U(1, 10)$  and

$b \sim U(10, 100)$ . The estimation of the hierarchical Bayesian model was done for the study farm sub-samples from single countries.

Two alternative specifications of the regression model—a linear and a quadratic model—were employed. In both the copula-based and regression approaches, the yield strike level was set to the expected value of the farm yield. The evaluation of the insurance contract risk reduction was done by measuring risk reductions for catastrophic years which were determined in two alternative ways: (i) by detecting years with lowest yield observations, i.e. based on the farm yield distribution; (ii) by using weather index to identify extreme drought years, i.e. based on the weather index distribution. Accordingly, three above specified risk measures were calculated for the 10, 20, and 30% left-tail realizations of the unconditional and conditional farm yield distributions. The evaluation of the insurance contracts with weather index thresholds equal to the first, second, and third deciles of the weather index distribution was done with respect to three above-mentioned parts of the unconditional and conditional yield distributions, respectively.

## **PRELIMINARY RESULTS**

The estimates of the dependence parameter of the survival Gumbel copula (Figure A1 in Appendix) indicate a solid level of dependence of the study farm yields on the selected drought index. For almost all study farms, the estimates of the dependence parameter are above 1.5. However, the highest degree of dependence was estimated for the farms in the first county, where the dependence parameter estimates are above 2.0 for the majority of the farms, while the lowest level of dependence on average was found for the farms in county 3.

Table 1 summarizes the estimates of the risk reduction for single weather index insurance contracts. Independent of the approach used for contract rating, as well as the choice of the weather index threshold level, risk reduction evaluated in terms of certainty equivalent is almost negligible. This result is not surprising considering the independence assumption of the expected utility model, which implies that risk preferences are linear in probabilities. Accordingly, relatively small changes in the left tail of the outcome distribution should not have a substantial influence on the expected utility value.

The estimates of risk reduction based on the expected shortfall criterion show that a substantially higher risk reduction was found for the copula-based approach than for the regression-based approach. On average, for the whole sample, the copula-based insurance contract with the threshold equal to the first decile of the weather index distribution would increase expected yield of the lowest 10% yield records by 79%.

**Table 1.** Relative risk-reduction estimates for copula-based approach and regression-based approach (3 weather index thresholds): catastrophic years determined based on farm yield distribution

	copula approach			linear regression model			quadratic regression model		
	q(PDI)=0.1	q(PDI)=0.2	q(PDI)=0.3	q(PDI)=0.1	q(PDI)=0.2	q(PDI)=0.3	q(PDI)=0.1	q(PDI)=0.2	q(PDI)=0.3
<i>Certainty equivalent</i>									
county 1	0.019	0.013	0.020	0.019	0.019	0.024	0.018	0.018	0.022
county 2	0.003	0.000	-0.001	0.006	0.006	0.007	0.003	0.003	0.005
county 3	0.013	0.012	0.012	0.012	0.012	0.013	0.007	0.007	0.008
county 4	0.004	0.003	0.004	0.005	0.005	0.008	0.005	0.005	0.007
county 5	0.007	0.019	0.015	0.009	0.009	0.016	0.006	0.006	0.011
whole sample	0.009	0.008	0.010	0.010	0.010	0.014	0.008	0.008	0.011
<i>Expected shortfall</i>									
county 1	1.496	0.574	0.450	1.233	0.530	0.338	0.987	0.424	0.290
county 2	0.552	0.285	0.205	0.399	0.173	0.117	0.128	0.061	0.055
county 3	0.704	0.348	0.182	0.626	0.262	0.129	0.317	0.124	0.058
county 4	0.418	0.257	0.254	0.212	0.105	0.120	0.179	0.092	0.112
county 5	0.560	0.579	0.349	0.323	0.196	0.184	0.174	0.100	0.104
whole sample	0.790	0.407	0.298	0.594	0.266	0.186	0.393	0.176	0.135
<i>Lower partial moment</i>									
county 1	0.347	0.310	0.451	0.343	0.348	0.430	0.309	0.306	0.390
county 2	0.160	0.116	0.175	0.208	0.227	0.267	0.109	0.134	0.159
county 3	0.320	0.327	0.307	0.295	0.314	0.304	0.154	0.159	0.150
county 4	0.164	0.196	0.318	0.187	0.233	0.304	0.173	0.218	0.286
county 5	0.209	0.548	0.509	0.224	0.439	0.422	0.140	0.283	0.263
whole sample	0.238	0.277	0.346	0.253	0.304	0.346	0.186	0.223	0.261

*Source:* author's estimates

The corresponding values for the hypothetical insurance contracts based on the linear and quadratic regression models are 59 and 39%, respectively. Risk reduction reduces for contracts with higher threshold levels of the weather index. Additionally, insurance contracts based on the linear regression model specification are found to provide higher risk reductions than those corresponding with the quadratic model. Consequently, in the following we reduce our discussion to the comparison of results obtained from the copula-based insurance approach and the linear regression model. The results of the t-test for the difference between two sample means indicate that the copula-based weather index contracts with the threshold levels equal to  $q_{0.2}(W)$  and  $q_{0.3}(W)$  would allow for a significantly higher average risk reduction (at the 5% level of significance) than corresponding contracts derived on the basis of the linear

regression; for the insurance contracts with the  $q_{0.1}(W)$  threshold the difference between average risk reductions from two alternative approaches is not significant.

Though our estimates of the lower partial moment suggest a solid reduction in the variability of farm yields in the left tail of the yield distribution, there are no significant differences in the average risk reductions between the contracts based on the copula approach and those derived by means of the linear regression model according to the two-sample mean t-test. Obviously, because they provide higher levels of coverage for the yield losses caused by extreme drought events, the copula-based contracts charge higher premiums; also in the years when an extreme yield loss was caused by another peril and thus no indemnity was paid by the specified insurance contract. This might be an explanation for a relatively moderate performance of copula-based insurance contracts as evaluated by using the lower partial moment.

**Table 2.** Relative risk reduction estimates for copula-based approach and regression-based approach (3 weather index thresholds): catastrophic years determined based on weather index distribution

	copula approach			linear regression model			quadratic regression model		
	$q(PDI)=0.1$	$q(PDI)=0.2$	$q(PDI)=0.3$	$q(PDI)=0.1$	$q(PDI)=0.2$	$q(PDI)=0.3$	$q(PDI)=0.1$	$q(PDI)=0.2$	$q(PDI)=0.3$
<i>Expected shortfall</i>									
county 1	1.906	0.791	0.610	1.601	0.361	0.384	0.424	0.290	0.893
county 2	0.811	0.591	0.462	0.504	0.197	0.202	0.061	0.055	0.873
county 3	0.957	0.494	0.335	0.689	0.205	0.196	0.105	0.058	0.849
county 4	0.756	0.519	0.405	0.377	0.142	0.187	0.092	0.112	0.824
county 5	1.062	0.825	0.508	0.590	0.252	0.237	0.100	0.104	0.945
whole sample	1.142	0.650	0.479	0.800	0.237	0.251	0.384	0.174	0.137
<i>Lower partial moment</i>									
county 1	0.572	0.615	0.873	0.520	0.585	0.754	0.309	0.306	0.390
county 2	0.391	0.582	0.849	0.447	0.571	0.694	0.109	0.134	0.159
county 3	0.627	0.749	0.824	0.541	0.628	0.670	0.169	0.167	0.150
county 4	0.375	0.471	0.945	0.329	0.449	0.617	0.173	0.218	0.286
county 5	0.283	0.940	0.979	0.314	0.601	0.623	0.140	0.283	0.263
whole sample	0.449	0.643	0.893	0.435	0.562	0.679	0.187	0.223	0.259

*Source:* author's estimates

An additional cause of the relatively moderate performance of the weather index insurance contracts might be explained by the capacity of the weather index to signal extreme drought occurrence. The estimates of risk reduction presented in Table 1 are obtained by employing the yield unconditional distribution. In this case, left-tail quantiles of the yield distribution may not necessarily correspond with left-tail quintiles of the weather index. If the number of disconcording pairs of joint yield and weather index realizations is not negligible, the evaluation of the insurance contract



effectiveness should account for this fact. In Table 2, we summarize risk reduction estimates obtained by employing yield realizations, conditioned of the weather index distribution quantiles. These estimates show what would be the risk reduction, if the selected weather index would perfectly identify the occurrence of drought and there were no other risk causing extreme yield losses. In this case, the copula-based insurance contracts would clearly outperform the contracts derived by employing the linear regression model. The t-test indicates that the average risk reductions due to the copula-based contracts with the  $q_{0.2}(W)$  and  $q_{0.3}(W)$ -thresholds would be significantly higher at the 1% level than the average risk reduction for equivalent contracts based on the linear regression as measured by both criteria - expected shortfall and lower partial moment. The difference in the average risk reduction between two alternative approaches for the contracts with the  $q_{0.1}(W)$  threshold is found to be significant at the 5% level for the expected shortfall and not significant for the lower partial moment. The highest risk reduction, in terms of an increase of the expected shortfall, were obtainable by setting the weather index threshold to correspond with the first quantile of its distribution, while higher reductions in the variability of the outcome would be achieved at higher levels of the index threshold.

## CONCLUSIONS

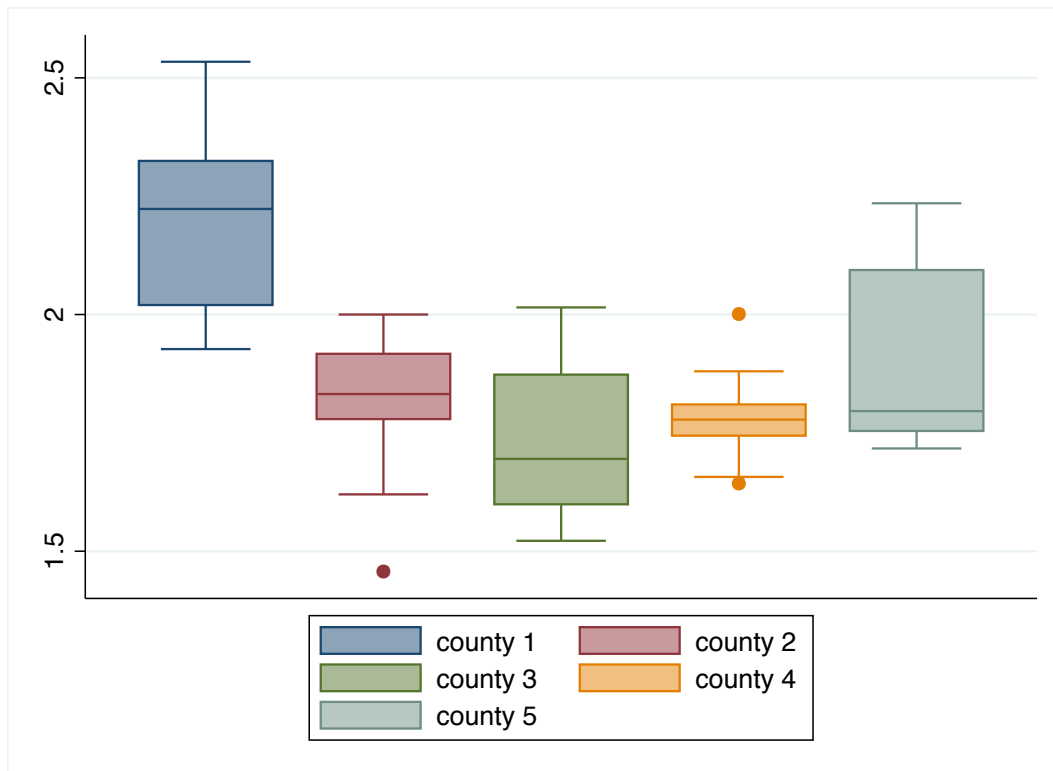
The paper presents a copula-based approach for rating weather index insurance designed to provide coverage for yield losses due to extreme weather events. The effectiveness of this approach is compared with the common regression-based approach. Our preliminary results suggest that the application of the copula approach might improve the performance of weather index insurance. However, the identification of drought seems to be problematic. Therefore, the selection of an adequate weather indicator to signal occurrence of an extreme event is a precondition for developing effective weather index insurance.

## REFERENCES

- Acharya, V., Pedersen, L. H., Philippon, T., and M. Richardson, M. (2012). Measuring systemic risk. CEPR Discussion Papers 8824, C.E.P.R. Discussion Papers.
- Akter, S., and Fatema, N. (2011). The role of microcredit and microinsurance in coping with natural hazard risks. Contributed paper, 18th Annual Conference of the European Association of Environmental and Resource Economists, Rome, June 29-July 2, 2011, Rome.
- Barnett, B., and Mahul, O. (2007). Weather Index Insurance for Agriculture and Rural Areas in Lower-Income Countries. *Am. J. Agr. Econ.* 89 (5): 1241-1247.
- Bokusheva, R. (2011). Measuring dependence in joint distributions of yield and weather variables. *Agricultural Finance Review*, 71:120-141.
- Bokusheva, R., and Breustedt, G. (2012). The effectiveness of weather-based index insurance and area-yield crop insurance: How reliable are ex post predictions for yield risk reduction? *Quarterly Journal of International Agriculture*, 51 (2):135-9156.
- Breustedt, G., R. Bokusheva, R., and O. Heidelbach, O. (2008): The potential of index insurance schemes to reduce farmers' yield risk in an arid region. *Journal of Agricultural Economics* 59 (2): 312-328.
- Cole, S., X. Giné, X., J. Tobacman, J., P. Topalova, P., R. Townsend, R., and J. Vickery, J. (2013). Barriers to Household Risk Management: Evidence from India. *American Economic Journal: Applied Economics*, 5 (1): 104-35.
- Gamerman, D., and Lopes, H.F. (2006). *Markov Chain Monte Carlo: Stochastic Simulation for Bayesian Inference*, 2nd ed. Chapman & Hall, London: Chapman & Hall.
- Genest, C., B. Remillard, B., and D. Beaudoin, D. (2009). Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and Economics*, 44: 199-214.
- Hardaker J.B., Huirne, R.B.M., Anderson, J.R., and Lien, G. (2007). *Coping with risk in agriculture*. 2nd edition. Wallingford: CABI Pub, Wallingford.
- Mainik, G., and Schaanning, E., (2012). On dependence consistency of CoVaR and some other systemic risk measures. RiskLab, Department of Mathematics, ETH Zurich (accessed at: <http://arxiv.org/pdf/1207.3464v3.pdf>).

- McNeil A., Frey R., and Embrechts, P. (2005). *Quantitative Risk Management*. Princeton: Princeton University Press, Princeton, 2005.
- Mobarak, A.M., and Rosenzweig, M. (2012). Selling Formal Insurance to the Informally Insured. Yale University Economic Growth Center Discussion Paper No. 1007.
- Nelsen, R. B. (1999). *An Introduction to Copulas*. Springer, New York: Springer.
- Norton, M., Holthaus, E., Madajewicz, M., Osgood, D., Peterson, N., Mengesha Gebremichael, C., Mullally, C., and Teh, T.L. (2011). Investigating Demand for Weather Index Insurance: Experimental Evidence from Ethiopia. Selected Paper prepared for presentation at the Agricultural & Applied Economics Association's 2011 AAEA & NAREA Joint Annual Meeting, Pittsburgh, Pennsylvania, July 24-26 July.
- Skees, J.R., Black, J.R., and Barnett, B.J. Barnett (1997). Designing and Rating an Area Yield Crop Insurance Contract. *American Journal of Agricultural Economics*, 79: 430-438.
- Skees, J., Murphy, A., Collier, B., McCord, M.J., and Roth, J. (2007). Scaling Up Index Insurance, What is needed for the next big step forward? Microinsurance Centre, LLC, Global Ag Risk, Inc.
- Sklar, A. 1959. Fonctions de répartition à n dimension et leurs marges, *Publ. Inst. Stat. Univ. Paris*, 1959; 8:299-231.
- Vedenov, D.V. and Barnett, B.J. Barnett (2004). Efficiency of weather derivatives as primary crop insurance instruments. In: *Journal of Agricultural and Resource Economics* 29 (3): 387-403.
- Wang, H. H., R. N. Karuaihe, R., Young, D. L., Young and Y. Zhang, Y. (2013). Farmers' demand for weather-based crop insurance contracts: the case of maize in South Africa. *Agrekon* 52 (1): 87-110.

## Appendix



Source: author's estimates

**Figure A1.** Estimates of survival Gumbel copula dependence parameter